Optimal photon pairs for quantum communication

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1. Abstract

The temporal wavepackets of photons emitted by means of spontaneous parametric down-conversion (SPDC) depend on the properties of a utilized pump laser and a nonlinear crystal. Here we derive analytical formulas allowing one to minimize the widths of such wavepackets for any quantum communication application, when the SPDC photons propagate through dispersive media. We show an example of a quantum key distribution scheme, which security distance can be extended by several tens of kilometres using our approach.

2. Temporal widths of SPDC photons

Spectral wavefunction of a pair of photons generated in SPDC process [1]:

$$\phi(\nu_1,\nu_2) = N\exp\left(-\frac{(\nu_1-\nu_2)^2}{\sigma^2} - \frac{(\nu_1+\nu_2)^2\tau_p^2}{4}\right), \quad (1)$$

where

ν_1, ν_2 – detunings from the central frequency

σ – effective phase-matching function width [2]

τ_p – pump laser pulse duration

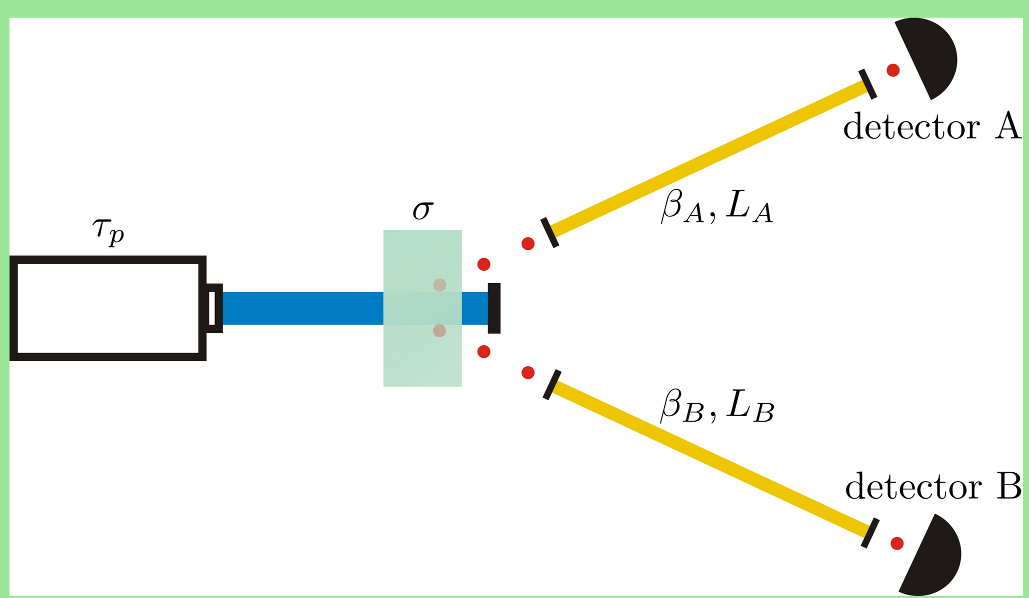

Figure 1: *Detection scheme for propagated SPDC photons. L_A and L_B are the lengths of the fibers, while their respective group velocity dispersion (GVD) values are equal to $2\beta_A$ and $2\beta_B$.*

Temporal width of the photon A when the **emission time** of pump laser pulse is **known**, but the **detection time** of the other photon is **unknown** (see [3, 4]):

$$\tau_A = \frac{\sqrt{\left(\tau_p^2 + D_A^2\sigma^2\right)\left(\sigma^2\tau_p^2+4\right)}}{2\sigma\tau_p}, \quad (2)$$

where $D_X = \beta_X L_X$

Temporal width of the photon A when **both** the **emission time** of pump laser pulse and the **detection time** of the other photon are **known**:

$$\tau_{Ah} = \sqrt{\frac{16\left(\tau_p^2 - D_A D_B\sigma^2\right)^2 + (D_A+D_B)^2\left(\sigma^2\tau_p^2+4\right)^2}{4\left(\tau_p^2+D_B^2\sigma^2\right)\left(\sigma^2\tau_p^2+4\right)}}. \quad (3)$$

Temporal width of the photon A when the **emission time** of pump laser pulse is **unknown**, but the **detection time** of the other photon is **known**:

$$\tau_{Ah,\Delta t} = \frac{\sqrt{16\tau_p^2 + 4\sigma^2\left(D_A-D_B\right)^2 + \sigma^4\tau_p^2\left(D_A-D_B\right)^2}}{2\sigma\tau_p}. \quad (4)$$

3. Optimizing SPDC source over the pump pulse duration

Analytical formulas for the symmetric scheme ($D_A = D_B \equiv \beta L$):

- Optimal pump pulse duration: $\tau_p^{\rm opt} = \sqrt{2|\beta|L}$
- Minimal temporal width of the photon A:
  – unheralded case: $\tau_A^{\rm min} = \frac{2+|\beta|L\sigma^2}{2\sigma}$
  – heralded case: $\tau_{Ah}^{\rm min} = 2\sqrt{\frac{\beta^2L^2\left(\beta^2L^2\sigma^4+4\right)}{\left(\beta^2L^2\sigma^2+2|\beta|L\right)\left(|\beta|L\sigma^2+2\right)}}$

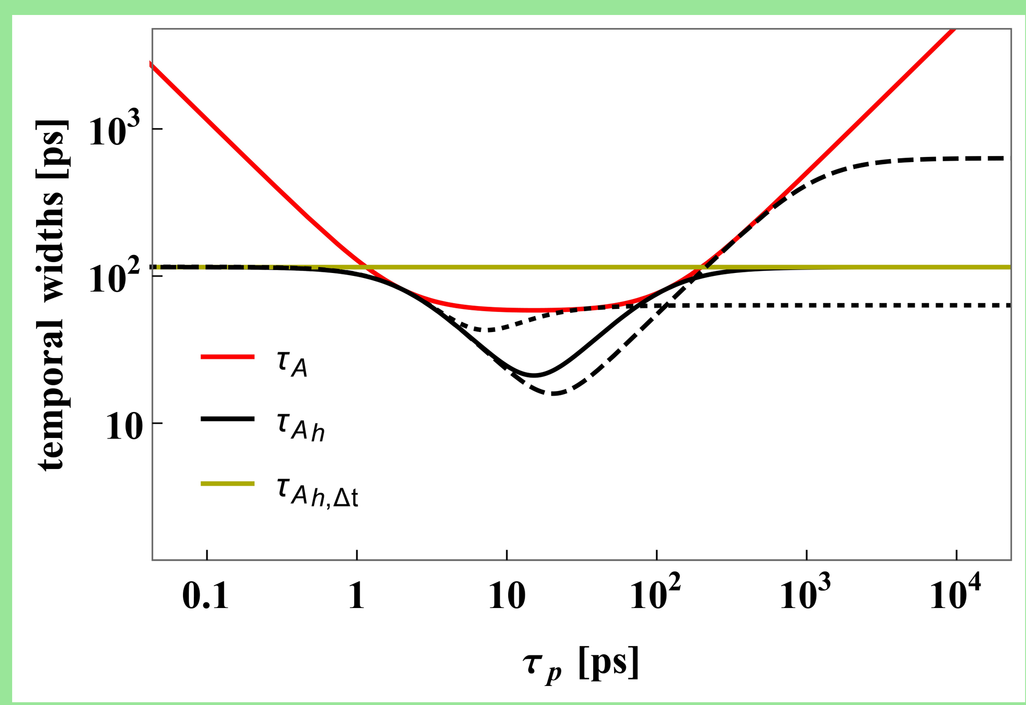

Figure 2: *Temporal widths of the photon entering the detector A plotted as a function of τ_p for $\sigma = 1\,\mathrm{THz}$, $\beta_A = \beta_B = -1.15\times10^{-26}\,\mathrm{s^2/m}$ (typical SMF fibers) and $L_A = 10\,\mathrm{km}$. Dotted, solid and dashed lines are plotted for $L_B = 1\,\mathrm{km}$, $L_B = 10\,\mathrm{km}$ and $L_B = 100\,\mathrm{km}$ respectively.*

4. Designing the optimal SPDC source

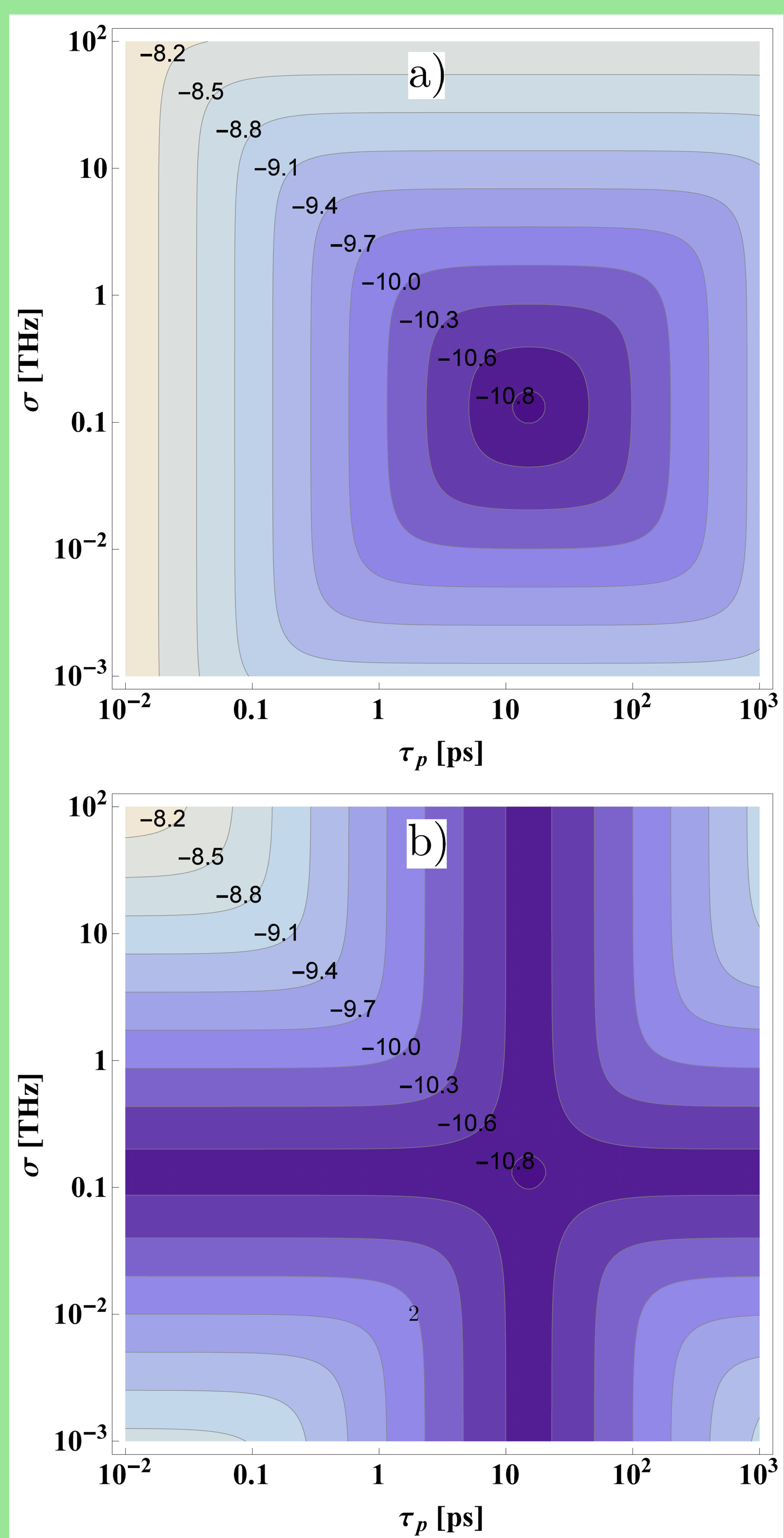

Figure 3: *Logarithm of temporal widths a) τ_A, b τ_{Ah} plotted as a function of τ_p and σ for $\beta_A = \beta_B = -1.15\times10^{-26}\,\mathrm{s^2/m}$ and $L_A = L_B = 10\,\mathrm{km}$.*

Analytical formulas for the symmetric scheme:

- Optimal effective phase-matching function width: $\sigma^{\rm opt} = \sqrt{\frac{2}{|\beta|L}}$
- Absolute minimum of the temporal width of the photon A: $\tau_A^{\rm abs} = \tau_{Ah}^{\rm abs} = \sqrt{2|\beta|L}$

5. Discussion

- Minimizing temporal width of photons entering the detectors is crucial to reduce the amount of noise registered during the detection process – so-called temporal filtering method.
- In the case of a given nonlinear crystal (fixed σ) neither very short nor very long pump laser pulses are optimal for reducing the temporal width of SPDC photons
- Temporal width of SPDC photons can be further minimized if one is able to freely choose the value of σ
- Future work: comparison between the requirements for optimal σ and the realistic range of the effective phase-matching function width

6. Application: improving the QKD security

Question: How much can we gain by optimizing the SPDC source for realistic quantum communication applications?

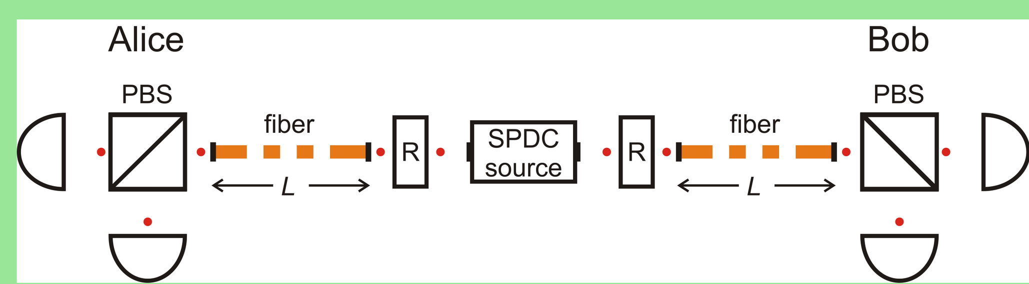

Figure 4: *QKD scheme with a SPDC source located in the middle of Alice and Bob. R denotes polarization rotators.*

Example: symmetric quantum key distribution with temporal filtering method used to reduce the detection noise (see [3] for detailed security analysis)

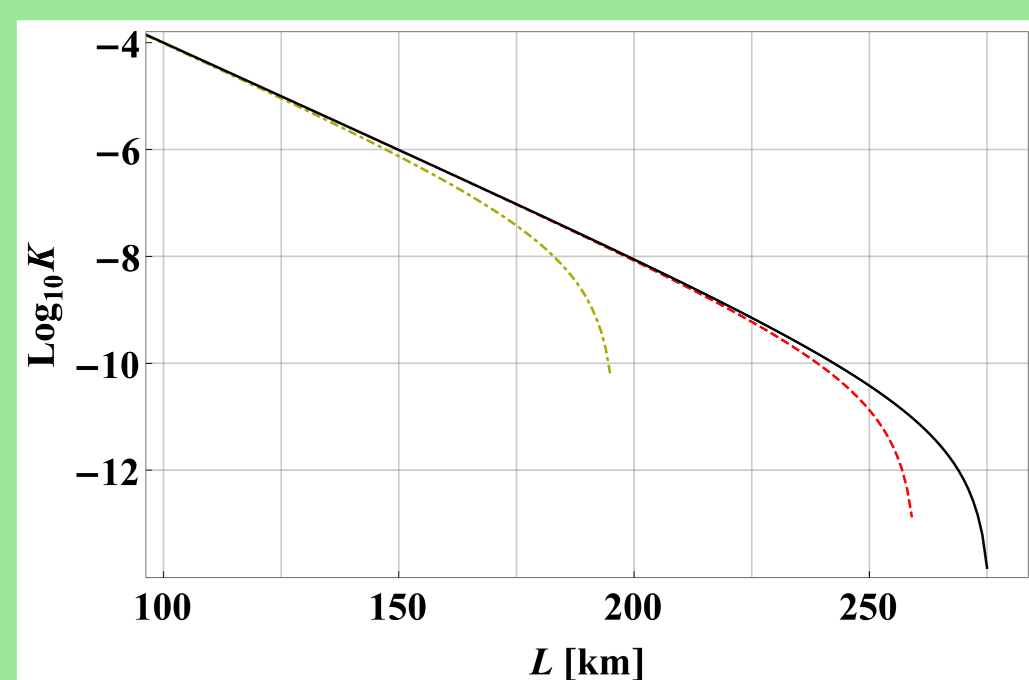

Figure 5: *Key generation rate for the symmetric QKD scheme plotted as a function of the length of the standard single-mode fibers used to connect the SPDC source with the participants of the BB84 protocol. The plots are made for the following cases: $\sigma = 1\,\mathrm{THz}$ and $\tau_p = 0.1\,\mathrm{ps}$ (yellow, dot-dashed line), $\sigma = 1\,\mathrm{THz}$ and $\tau_p = \sqrt{2|\beta|L}$ (red, dashed line), $\sigma = \sqrt{2/|\beta|L}$ and $\tau_p = \sqrt{2|\beta|L}$ (black, solid line).*

Conclusion: Optimizing SPDC source according to our guidance can extend the achievable maximal security distance by several tens of kilometers

References

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[2] P. Kolenderski, W. Wasilewski, K. Banaszek, Phys. Rev. A **80**, 013811 (2009).

[3] K. Sedziak, M. Lasota, P. Kolenderski, Optica **4**, 84 (2017).

[4] K. Sedziak, M. Lasota, P. Kolenderski, arXiv: 1711.06131 (2017).

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